LITERATURE CITED

1. U. L. Wilkinson, Non-Newtonian Fluids [Russian translation], Mir, Moscow (1964).
2. I. A. Charnyi, Principles of Subsurface Hydraulics [in Russian], Gostoptekhizdat, Moscow (1956).
3. A. A. Samarskii, Introduction to the Theory of Difference Methods [in Russian], Nauka, Moscow (1971).
4. G. Astarita and G. Marrucci, Principles of Non-Newtonian Mechanics, McGraw-Hill (1974).
5. A. V. Lykov, Heat Exchange [in Russian], Énergiya, Moscow (1972).

STABILITY OF A CONVECTIVE FLOW OF A VISCOUS FLUID BY THE METHOD OF LOCAL POTENTIAL

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UDC 532.516

A study is made of the stability of convective fluid flow caused by an external temperature gradient and heat sources uniformly distributed in the fluid.

Interest has recently been increasing in the study of convective fluid flow caused by internal heat sources. The physical mechanism of heat liberation may vary in different cases: Joulian dissipation, radiant heat transfer, absorption of external radiation, etc. Together with this, the results in [1, 2] showed the destabilizing effect of viscosity nonuniformity. It is naturally of interest to investigate the stability of convective motion caused both by an external temperature gradient and internal heat liberation in considering the temperature dependence of viscosity. For this purpose, we will examine the convective flow of a viscous fluid in a long vertical layer bounded by parallel surfaces $x = \pm d$ maintained at fixed temperatures $T = \pm\theta$. Let internal heat sources with a constant volume density q be uniformly distributed throughout the volume of the liquid. We will assume that the viscosity of the liquid depends on the temperature according to the linear law,

$$\nu = \nu_0 (1 - \alpha T), \tag{1}$$

where ν_0 is the maximum value of viscosity reached on the cold ($T = -\theta$) surface; α, temperature coefficient.

We will adopt the variational approach to study the stability of the convective flow — specifically, the method of local potential. We will construct a functional having certain extreme properties and dependent on two types of variables [3]. In accordance with [4], we will proceed on the basis of linearized equations of a perturbed state [1]

$$\frac{\partial u_x'}{\partial t} + G\bar{u}_z \frac{\partial u_x'}{\partial z} = -\frac{\partial p'}{\partial x} + \bar{\eta}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right) u_x' + 2\frac{\partial \bar{\eta}}{\partial x}\,\frac{\partial u_x'}{\partial x} + \frac{\partial \eta'}{\partial z}\,\frac{\partial \bar{u}_z}{\partial x}, \tag{2}$$

$$\frac{\partial u_z'}{\partial t} + G\left[\bar{u}_z \frac{\partial u_z'}{\partial z} + u_x' \frac{\partial \bar{u}_z}{\partial x}\right] = -\frac{\partial p'}{\partial z} + \bar{\eta}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right) u_z' + \eta' \frac{\partial^2 \bar{u}_z}{\partial x^2} + \frac{\partial \bar{\eta}}{\partial x}\,\frac{\partial u_z'}{\partial x} + \frac{\partial \eta'}{\partial x}\,\frac{\partial \bar{u}_z}{\partial x} + \frac{\partial \bar{\eta}}{\partial x}\,\frac{\partial u_x'}{\partial z} + T', \tag{3}$$

$$\frac{\partial T'}{\partial t} + G\left[u_x' \frac{\partial \bar{T}}{\partial x} + \bar{u}_z \frac{\partial T'}{\partial x}\right] = \frac{1}{\Pr}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right) T', \tag{4}$$

$$G = g\beta q d^5/2\nu_0^2 \rho c_p \chi; \quad \Pr = \nu_0/\chi; \quad \gamma = \alpha q d^2/2\rho c_p \chi; \quad \eta = 1 - \gamma T$$

with the boundary conditions

$$\bar{u}_z(\pm 1) = 0, \quad \bar{T}(\pm 1) = N, \quad u_x'(\pm 1) = u_z'(\pm 1) = T'(\pm 1) = 0.$$

Here the units of distance, time, velocity, temperature, and pressure are, respectively: d; d^2/ν_0; $g\beta q d^4/2\nu_0 \rho c_p \chi$; $qd^2/2\rho c_p \chi$; $g\beta q d^3/2c_p \chi$; $N = G_\Theta/G$, where $G_\Theta = g\beta\Theta d^3/\nu_0^2$; u_x, u_z, projections of the perturbed velocities on the x and z axes; p', T', pressure and temperature perturbations; $\bar{T}$ and $\bar{u}_z$, their mean values. Having performed all mathematical operations similar to the

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manner in [5], we obtain an expression for the local potential

$$\Phi=\int_{-1}^{1}dx\left\{\frac{\bar{\eta}}{2}\left[\left(\frac{d^2\psi}{dx^2}\right)^2+2k^2\left(\frac{d\psi}{dx}\right)^2+k^4\psi^2\right]+(\lambda+ikG\bar{u}_z)Q^0Q-ik\psi^0Q\frac{d\bar{T}}{dx}+\frac{1}{2}\left[k^2Q^2+\left(\frac{dQ}{dx}\right)^2\right]-\right.$$

$$\left.-\left[\frac{d\eta^0}{dx}\frac{d\bar{u}_z}{dx}+\eta^0\frac{d^2\bar{u}_z}{dx^2}\right]\frac{d\psi}{dx}+\left[(\lambda+ikG\bar{u}_z)\left(k^2-\frac{d^2\psi^0}{dx^2}\right)+\frac{d^2\bar{\eta}}{dx^2}k^2\psi^0+\eta k^2\frac{d\bar{u}_z}{dx}+ikG\psi^0\frac{d^2\bar{u}_z}{dx^2}+\frac{dQ^0}{dx}\right]\psi\right\}. \tag{5}$$

Here the perturbations are chosen in the form

$$u_x'=-ik\psi\exp(\lambda t+ikz),\quad u_x^{0\prime}=-ik\psi^0\exp(\lambda t+ikz),$$

$$u_z'=\frac{d\psi}{dx}\exp(\lambda t+ikz),\quad u_z^{0\prime}=\frac{d\psi_0}{dx}\exp(\lambda t+ikz),$$

$$T'=Q\exp(\lambda t+ikz),\quad T^{0\prime}=Q^0\exp(\lambda t+ikz),$$

$$p'=h\exp(\lambda t+ikz),\quad p^{0\prime}=h^0\exp(\lambda t+ikz),$$

where ψ and Q, h are certain functions of the variable x; k, wave number; $\lambda = \lambda_r + i\lambda_i$, complex decrement, while the superscript 0 pertains to invariant quantities.

The temperature and velocity for unperturbed motion entering into functional (5) were found earlier in [6] and are determined as follows:

$$\bar{T}=Nx+1-x^2,\quad \bar{u}_z=\frac{1}{6\gamma}\left\{x^2-1-N(x+1)+[J_0(x)-J_0(-1)]C+Z\ln\frac{F(x)}{F(-1)}\right\}.$$

Here

$$F(x)=\gamma x^2-\gamma Nx+1-\gamma;\quad \Delta=4\gamma(1-\gamma)-\gamma^2N^2;$$

$$C=-\left\{2N+Z\ln\frac{F(-1)}{F(1)}\right\}\{J_0(-1)-J_0(1)\}^{-1};$$

$$Z=-\left\{\frac{4+3N^2}{3}-\frac{N\ln\frac{F(-1)}{F(1)}}{\gamma[J_0(-1)-J_0(1)]}\right\}\times$$

$$\times\left\{\frac{1}{2\gamma}\left[\ln\frac{F(-1)}{F(1)}\right]^2[J_0(-1)-J_0(1)]^{-1}+4+\frac{\Delta}{2\gamma}[J_0(-1)-J_0(1)]\right\}^{-1};$$

$$J_0(x)=\frac{2}{\sqrt{\Delta}}\operatorname{arctg}\frac{2\gamma x-\gamma N}{\sqrt{\Delta}},\quad 4\gamma(1-\gamma)>\gamma^2N^2;$$

$$J_0(x)=\frac{2}{\sqrt{-\Delta}}\ln\frac{2\gamma x-\gamma N-\sqrt{-\Delta}}{2\gamma x-\gamma N+\sqrt{-\Delta}},\quad \gamma^2N^2>4\gamma(1-\gamma)$$

and $F(\pm1)$, $J_0(\pm1)$ are the values of F(x) and $J_0(x)$ at the boundary points $x = \pm1$.

Since the flow being studied consists of opposite convective currents [6], there are grounds for reasoning that its critical value (Re_{cr}) is connected with a hydrodynamic mechanism [7]. Thus, the stability of such a fluid motion should be determined in a purely hydrodynamic formulation, ignoring thermal perturbations Q and their effect on the development of hydrodynamic perturbations. We should note that such an approach is valid within the region of low Prandtl numbers. Let us set $Q=\frac{dQ}{dx}=\frac{d^2Q}{dx^2}=0$, in (5) and choose test functions for ψ in the form

$$\psi=\sum_{i=0}^{2}A_ix^i(1-x^2)^2,\quad \psi^0=\sum_{i=0}^{2}A_i^0x^i(1-x^2)^2. \tag{6}$$

Substituting (6) in Eq. (5), satisfying the conditions of stationariness

$$\left(\frac{\partial\Phi}{\partial A_0}\right)_{A_0^0A_1A_1^0A_2A_2^0}=0,\quad \left(\frac{\partial\Phi}{\partial A_1}\right)_{A_0A_0^0A_1^0A_2A_2^0}=0,\quad \left(\frac{\partial\Phi}{\partial A_2}\right)_{A_0^0A_0A_1A_1^0A_2^0}=0,$$

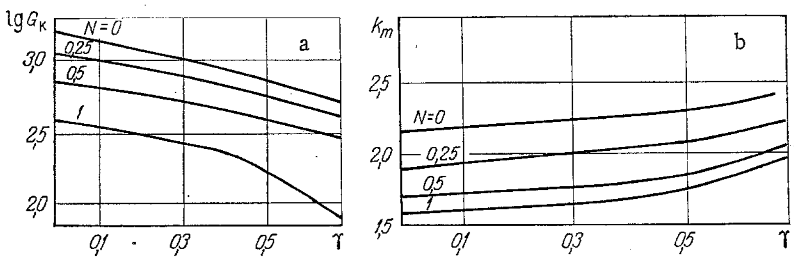

Fig. 1. Dependence of critical Grashof numbers G_k (a) and critical wave number k_m (b) on degree of nonuniformity of viscosity γ for different N.

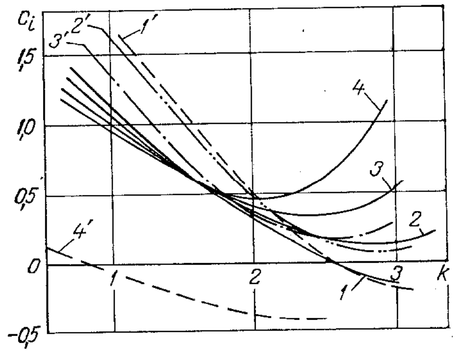

Fig. 2. Phase velocities of neutral perturbations $c_i = (\lambda_i/kG)\cdot 10^{-2}$ for different γ and N: 1) N = 0; 2) 0.25; 3) 0.5; 4) 1 at γ = 0. The dashed lines 1'-4' are for γ = 0.5.

and using the auxiliary conditions $A_0^0 = A_0$, $A_1^0 = A_1$, $A_2^0 = A_2$ for a neutral perturbation regime where $\lambda_r = 0$, with $\lambda_i = f(\gamma, N, k, G)$, we obtain the following expression

$$y^3L_3 + y^2L_2 + yL_1 + L_0 = 0, \tag{7}$$

where $y = k^2G^2$, and L_0, L_1, L_2, L_3 are expressed in complex form through integrals of $\bar{u}_z$ and $\bar{T}$ and power series in x. It should be noted that Eq. (7) makes it possible to determine the critical Grashof numbers G_k and critical values of the wave vector k_m satisfying the condition of neutral stability.

Let us discuss the results obtained. Figure 1a shows the dependence of the critical Grashof numbers G_k on the degree of nonuniformity of the viscosity γ for different N. It is apparent that an increase in γ at any N leads to a decrease in G_k, i.e., to a reduction in the stability of convective flow. Meanwhile, the lower N, the higher lies the corresponding curve. It should be noted that with a constant viscosity γ = 0 in the boundary layer and the absence of an external temperature gradient (N = 0), we obtained a critical Grashof number G_k = 1651 for the test functions (6), while G_k = 1712 if the basis functions are chosen in the form of the amplitudes of perturbations in a quiescent liquid [8]. This agrees fairly well with the results in [7] (G_k = 1720), where the authors employed the Galerkin method, using 16 basis functions.

As concerns the wave numbers k_m (Fig. 1b), with an increase in γ they shift in the direction of larger k, i.e., into the short-wave region. If there are no internal heat sources, then an increase in viscosity nonuniformity is accompanied, in accordance with the results in [2], by a shift of the minimum of the neutral curve for hydrodynamic-type perturbations in the direction of long-wave perturbations.

As was shown in [6], the structure of the velocity profile depends considerably on the value of N. For N = 0, e.g., accounting for viscosity nonuniformity leads only to an increase in the flow rate, without changing its symmetry. The velocity profile begins to be restructured with an increase in N (becoming two streams instead of three) and, apart from an increase in the absolute value of the profile, it also becomes asymmetrical. The latter obviously leads to a more unstable fluid motion.

The features of the unperturbed profile $\bar{u}_z$ also affect the phase velocities of neutral perturbations. For N = 0 for example in the case of constant viscosity, the phase velocity changes sign with a change in the parameters along the neutral curve (Fig. 2) so that the point k = 2.61 (versus k = 2.65 in [7]) corresponds to a neutral "standing" perturbation. With an increase in γ, standing perturbations are shifted into the shortwave region. For example, at $\gamma = 0.5$ and N = 0, k = 2.62. For other values of $N \neq 0$, perturbations with a phase velocity equal to zero cannot exist (Fig. 2), i.e., the perturbations drift along the flow. We might point out the oddness of the profile in the special case N = 1 and $\gamma = 0.5$ [6], and "standing" perturbations are again possible for k = 0.82.

NOTATION

p, convective pressure reckoned from the hydrostatic pressure at the mean density ρ; ν, kinematic viscosity coefficient; β, χ, coefficient of linear expansion and diffusivity, assumed constant; g, acceleration due to gravity; c_p, specific heat.

LITERATURE CITED

1. G. Z. Gershuni, "On the stability of steady-state convective flow of a viscous liquid," Uch. Zap. Permsk. Gos. Univ., 19, No. 3, 25-28 (1961).
2. G. Z. Gershuni, E. M. Zhukhovitskii, and V. M. Shikhov, "On the stability of convective flow of a liquid with a viscosity dependent on temperature," Teplofiz. Vys. Temp., 13, No. 4, 771-778 (1975).
3. P. Glensdorff and I. Prigogine, Thermodynamic Theory of Structure, Stability, and Fluctuations [Russian translation], Mir, Moscow (1973), pp. 129-149.
4. P. Glansdorff and I. Prigogine, "On a general evolution criterion in macroscopic physics," Physica, 30, 351-374 (1964).
5. A. I. Lopushinskaya, V. V. Negrich, V. V. Nechiporuk, and S. V. Markova, "Method of local potential in determining the limits of stable convective flow," Zh. Tekh. Fiz., 45, No. 9, 1976-1980 (1975).
6. V. V. Gorlei, V. N. Kapranov, and V. A. Shenderovskii, Study of Convective Flow in a Vertical Channel. Steady-State Case. Kiev (1978), pp. 12-19 (Preprint/IF-78-15).
7. G. Z. Gershuni, E. M. Zhukhovitskii, and A. A. Yakimov, "On the stability of steady-state convective motion caused by internal heat sources," Prikl. Mat. Mekh., 34, No. 4, 700-705 (1970).
8. R. V. Birikh, G. Z. Gershuni, and E. M. Zhukhovitskii, "On the perturbation spectrum of plane-parallel flows at low Reynolds numbers," Prikl. Mat. Mekh., 20, No. 1, 88-98 (1965).

DEPENDENCE OF THE MASS TRANSFER DURING DISSOLUTION OF A ROUGH WALL IN A PLANE CHANNEL ON THE STRUCTURE OF THE STREAM

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A channel wall with a sinusoidally rough surface is considered and the location of the point on this surface where the diffusion current reaches its maximum is determined, depending on the Reynolds number as well as on the roughness wavelength and amplitude.

The equation of vortex transport for the flow function ψ and the boundary conditions for steady two-dimensional flow of a viscous incompressible fluid through a plane channel are

$$\frac{\partial^4\psi}{\partial x^4}+2\frac{\partial^4\psi}{\partial x^2\partial y^2}+\frac{\partial^4\psi}{\partial y^4}=\frac{1}{\nu}\left[\frac{\partial\psi}{\partial y}\frac{\partial\Delta\psi}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\Delta\psi}{\partial y}\right], \quad (1)$$

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